BACCALAURÉAT GÉNÉRAL ET TECHNOLOGIQUE
ÉPREUVE SPÉCIFIQUE DES SECTIONS EUROPÉENNES
MATHEMATIQUES – ANLGAIS

SUJET 7 – Square roots by compass-and-straightedge construction

Thème : Geometry

Ce sujet comporte 1 page. L'usage de la calculatrice est autorisé.

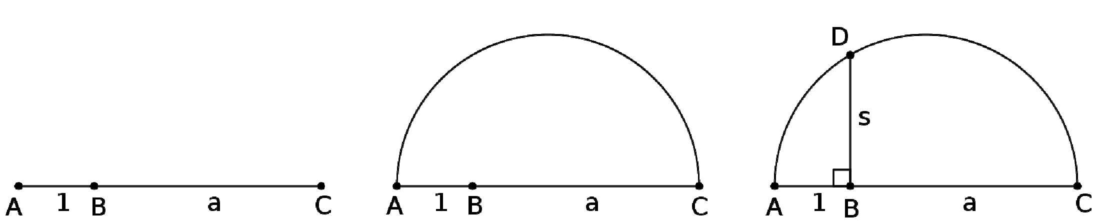
In antiquity, geometric constructions of figures and lengths were restricted to the use of only a straightedge and compass. Although the term "ruler" is sometimes used instead of "straightedge," the Greek prescription prohibited markings that could be used to make measurements.

Because of the prominent place Greek geometric constructions held in Euclid's Elements, these constructions are sometimes also known as Euclidean constructions. Such constructions lay at the heart of some geometric problems of antiquity such as circle squaring, cube duplication, and angle trisection. The Greeks were unable to solve these problems, but it was not until hundreds of years later that the problems were proved to be actually impossible under the limitations imposed.

The squaring of the circle asks to construct a square equal in area to a circle using only a straightedge and compass. To solve this problem, if the radius of the circle is set equal to one, you need to construct a length equal to the square root of π which was proved to be impossible by compass-and-straightedge in 1882 by Lindemann. But it is possible to construct a length equal to the square root of any whole number as we will see in the following exercise.

1. Dégager les idées essentielles du texte ci-dessus.

The aim of this exercise is to show that the step-by-step construction below, using a semi-circle, gives a length BD equal to the square root of the length BC.



2. Questions :

   a) Describe how you would construct each step of the above construction with compass-and-straightedge if a=3 and AB=1 is given.

   b) Using the Pythagorean theorem, express AD^2 in terms of s.

   c) Express DC^2 in terms of s and a.

   d) Explain why we can write : $1+s^2+a^2+s^2=(1+a)^2$

   e) Solve this equation for s. Have we proven what we wanted?