

**BACCALAURÉAT GÉNÉRAL ET TECHNOLOGIQUE
ÉPREUVE SPÉCIFIQUE DES SECTIONS EUROPÉENNES
MATHÉMATIQUES – ANGLAIS**

SUJET 10 – Pythagorean triples

Thème : Arithmetic and geometry

Ce sujet comporte 1 page. L'usage de la calculatrice est autorisé.

5 The Greeks did not express Pythagoras's theorem as an equation in the modern symbolic sense. That came later with the development of algebra. In ancient times, the theorem was expressed verbally and geometrically. It attained its most polished form, and its first recorded
10 proof, in writings of Euclid of Alexandria. Around 250 BC Euclid became the first modern mathematician when he wrote his famous Elements, the most influential mathematical textbook ever. Euclid turned geometry into logic by making his basic assumption explicit and invoking them to give systematic proofs for all of his theorems.
15 He built a conceptual tower whose foundations were points, lines, and circles, and whose pinnacle was the existence of precisely five regular solids.



15 One of the jewels in Euclid's crown was what we now call Pythagoras's theorem. In the famous translation by Sir Thomas Heath this proposition reads: "In right-angled triangles the square on the side subtending the right angle is equal to the squares on the sides containing the right angle."

(Extract from "17 Equations that changed the World" by Ian Stewart)

1. Dégager les idées essentielles du texte ci-dessus.

2. Questions mathématiques

A Pythagorean triple is a triple $(a; b; c)$ of whole numbers that fits the rule: $a^2 + b^2 = c^2$.

a) Check whether the four following triples are Pythagorean triples:

$(3; 4; 5)$ $(4; 5; 7)$ $(1; 1; \sqrt{2})$ $(-3; 4; -5)$

b) Explain how to draw a triangle ABC whose lengths are $AB=3$, $AC=4$ and $BC=5$ using a ruler and a pair of compasses.

c) What can you say about a triangle whose lengths are a Pythagorean triple?

d) Let's assume $(a; b; c)$ is a Pythagorean triple, prove that $(2a; 2b; 2c)$ is a Pythagorean triple as well. What does this result mean geometrically?

e) Does a Pythagorean triple exist such that all 3 numbers are odd numbers?